Fairness index in single and double star Networks

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Abstract

In wireless network, the communication works in half duplex mode and nodes can interfere together. In this context, fairness is not obvious. This paper will focus on fairness in the received packets by each node. Fairness is evaluated for static networks topologies called Single Star Network or Double Star Network. The fairness is quantified by its index. In this work, the evaluation of fairness index for double star network is given. Some value of this index are not possible for double star network topology. For example the index of one can only be possible if the double star network is simular to star network. Then the star networks are studied and some simulations are used to illustrate the way to get fairness in the network by controlling the flow rates.

1 Introduction

The performance of wireless 802.11 MAC protocol is generally evaluated with two parameters : collision probability and fairness [1],[13]. The fairness algorithm was widely studied by different research groups. Jain, Chiu and Hawe give us a mathematical definition in [6]. Their paper introduces the fairness index for any kind of resource sharing. This definition will be applied to the packet rates received by nodes.

TCP fairness was studied in [4], where the authors propose a distributed algorithm on neighbors to improve TCP fairness. In another paper [11] the authors propose a scheduling algorithm to get fairness in a multi-hop wireless network. Some papers are also based on the study of a distributed algorithm to maximalize throughput and fairness in Ad Hoc networks [2],[3] .

This paper is focused in fairness on node reception rates in an Ad-Hoc wireless network. A general introduction for fairness and fairness index is given. After that, a description of the network characteristic is done. For a theoretical analysis of the problem, the fairness index is evaluated for a basic network called Double Star Network and Star network. The existence of some value of fairness index is proved. We recall form [15] that exact fairness, such that fairness index is one, is not possible until Double Star Network degenerates in a Single Star Network. The fairness of Star network is studied and an algorithm is recalled from [14]. The simulation is done with NS2-2.33, will show that fairness can be accomplished by limiting the transmission rate of some nodes.

2 Network model and fairness

This work is done in the area of Ad-Hoc wireless network. An Ad-Hoc network is made of wireless nodes which establish wireless communications between themselves. In this context, there is no central infrastructure. This means that the nodes are equivalent. A node can be in two states at a given time, transmission or reception. The limitation of radio communications implies that the communications of a channel are limited in distance and they act in half duplex mode. These characteristics are described as follow :

2.1 Network model

We consider an Ad-Hoc network such as :

- The network is packet-switched
- The nodes are in half-duplex mode
- Only nodes in some distance can communicate
- The time is divided in time slots
- Packets are sent in time slots

2.2 Fairness index

In an Ad-Hoc network, the nodes have an equivalent role. Because the communications are in half-duplex mode, the position of the node in the network topology is dramatically related to its transmission rate. Some nodes with high degrees of connectivity in transmission will interfere with a high number of neighbors. This implies that the transmission and reception rates of the nodes are different.

In this paper, we try to give a fair access to each node. Fairness can be expressed as a mathematical formula given in [6]. The formula is based on a resource independent model and can be used to express fairness for any shared resource. It is also independent of network scalability. The resource will be applied to the reception rate of each node.

We recall the fairness index definition form [6] :

Definition 2.1. The fairness index of a shared resource x_i is given by

$$f(x_i) = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \sum_{i=1}^{n} x_i^2} \tag{1}$$

We will apply this approach to the reception rate of a node. Here are some of the notations :

Notation. The following notations will be used in this work :

- Let x_i be the reception rate of node X_i
- Let $r_{j,i}$ be the reception rate of node X_i of the packets send by X_j
- Let D_i be the degree of node X_i
- Let S_j^i be the transmission rate of node X_j which is a neighbor of node X_i

Remark 1. Using the previous notations, we have a reception rate x_i of node X_i :

$$x_i = \sum_{j=1}^{D_i} r_{j,i}$$

Node X_j has transmission rate S_j. Therefore we have $r_{j,i} = S_j^i$. This gives us :

$$x_i = \sum_{j=1}^{D_i} S_j^i \tag{2}$$

3 Double Star Network

Compute a fairness condition on reception rates is not easy for some Ad-Hoc networks. From a theoretical approach, the star double network and star network are introduced. This network is simple enough to compute a relation on transmission rates to get fairness. It can represent a subgraph of an Ad-Hoc network composed of a central nodes and its neighbors.

3.1 Definition

Definition 3.1 (Double Star network)**.** The double star network $SN_{n,m}$ is composed of $n + m + 1$ nodes $\{X_0, X_1, \ldots, X_n, X_{n+1}, ..X_{n+m}\}$ where :

- $\{X_i, i \leq n\}$ are neighbors of the node X_0, $\{X_0, X_i, n + 1 \leq i \leq n + m\}$ are neighbors of the node X_n
- and there is no connection between X_i, X_j for $i, j > 0$ only X_0 is the neighbor of X_n.

This is an example :

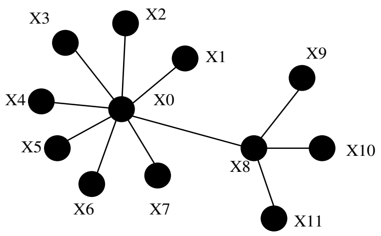

Figure 1. The $SN_{8,3}$ double star network

When the central nodes X_0 or X_n sends a packets, it will be received by their neighbors and this is not fair. Each set of packets respectively sent by X_i where $0 < i < n+1$ and or by X_j where $n < j < m+n+1$ will be seen respectively only once by X_0 or X_n.

3.2 Fairness index of the double star network

We can compute the fairness index for a double star network :

Lemma 3.1. *For a double star network $SN_{n,m}$, the fairness index α for the reception rate is given by the equation :*

$$2\left(2 - \alpha(n + m + 1)\right) X^2 - 2\alpha(n + m + 1)Y^2 + \mathcal{Q}(S_0, S_n) = 0 \tag{3}$$

where :

$$X = \sum_{i=1}^{n-1} S_i + \frac{(2n - \alpha(n+m+1))S_0}{2(2-\alpha(n+m+1))} + \frac{(2(m+1) - \alpha(n+m+1))S_n}{2(2-\alpha(n+m+1))}$$

$$Y = \sum_{i=n+1}^{n+m} S_i - \frac{\alpha(n+m+1)S_0 - \alpha(n+m+1)S_n}{2\alpha(n+m+1)}$$

$$\mathcal{Q}(S_0, S_n) = AS_0^2 + 2BS_0S_n + CS_n^2$$

is a quadratique form where

$$A = -\frac{(n-1)(n+m+1)\alpha((n+m+1)\alpha - n - 1)}{(n+m+1)\alpha - 2}$$

$$B = \frac{m(n-1)(n+m+1)\alpha}{\alpha(n+m+1) - 2}$$

$$C = -\frac{(n+m+1)^2\alpha^2(2m-3) - 2(n+m+1)\alpha(m^2-2)}{2(\alpha(n+m+1)-2)}$$

Proof. Let α be :

$$\alpha = f(x)$$

some basic computation gives the equation (3). □

The equation (3) might have no solutions. Let's see which value α will give no trivial solutions. Equation (10) is a quadratique relation in X and Y it has a constant term $\mathcal{Q}(S_0, S_n)$.

3.2.1 The coefficients of X^2 and Y^2

The coefficient of Y^2 is $-2\alpha(n+m+1)$ it is negative because α is a fairness index and therefore α is positive.

The coefficient of X^2 is $2(2 - \alpha(n+m+1))$. Its sign depends on $2 - \alpha(n+m+1)$.

The following lemma gives some results about the existence of solution of (3) :

Lemma 3.2. *For a double star network, the existence of fairness index α is submit to the following rules :*

- *If $\frac{2}{n+m+1} > \alpha$ then there exits no trivial solutions for equation (3).*
- *If $\alpha > \frac{2}{n+m+1}$ then there exists no trivial solutions for equation (3) if $\mathcal{Q}(S_0, S_n) > 0$.*

Proof. If $\frac{2}{n+m+1} > \alpha$ then the coefficient of X^2 and Y^2 have opposite sign. This implies the existence of solutions. If $\alpha > \frac{2}{n+m+1}$ then the coefficient of X^2 and Y^2 have same sign. To have no trivial solution, the quadratique form $\mathcal{Q}(S_0, S_n)$ has to be positive. □

The lemma 3.2 shows the importance of the sign of the quadratique form $\mathcal{Q}(S_0, S_n)$ if $\alpha > \frac{2}{n+m+1}$. In the next part, the sign of quadratique form will be studied.

3.2.2 The sign of the quadratique form $\mathcal{Q}(S_0, S_n)$

Recall that :

$$\mathcal{Q}(S_0, S_n) = AS_0^2 + 2BS_0S_n + CS_n^2 \tag{4}$$

In the lemma 3.2, the sign of the quadratique form $\mathcal{Q}(S_0, S_n)$ is important for $\alpha > \frac{2}{n+m+1}$. Let's suppose that $(n+m+1)\alpha - 2 > 0$, then the denominator of A, B and C are positive. Let's study their numerator :

- The sign of A is the sign of $n + 1 - (n+m+1)\alpha$.
- The sign of B is positive because $n > 1$.
- The sign of C is the sign of $2(m^2-2) - (n+m+1)\alpha(2m-3)$.

The next lemma will give conditions on n and m to have A, B and C positive.

Lemma 3.3. *If $\frac{2}{n+m+1} < \alpha$ and $m \geq 1$ then*

$$\frac{m^2-2}{2m-3} \geq 1$$

and C is positive if

$$\frac{2}{m+n+1} < \alpha \leq \frac{2(m^2-2)}{(n+m+1)(2m-3)}$$

Proof. If $\alpha > \frac{2}{n+m+1}\frac{m^2-2}{2m-3}$ then C is negative else C is positive. The sign of C is given by

$$2(m^2-2) - (n+m+1)\alpha(2m-3)$$

Let's prove that :

$$2(m^2-2) - (n+m+1)\alpha(2m-3) < 0$$

This implies that :

$$\frac{2}{n+m+1}\frac{m^2-2}{2m-3} < \alpha$$

□

Lemma 3.4. *A is positive if :*

$$\frac{2}{n+m+1} < \alpha \leq \frac{n+1}{n+m+1}$$

Proof. The sign of A is given by :

$$n + 1 - (n+m+1)\alpha$$

□

The next theorem will give a condition for the quadratique form $\mathcal{Q}(S_0, S_n)$ to be positive.

theorem 3.1. *If $1 < n \leq m$ and*

$$\frac{2}{n+m+1} < \alpha \leq \frac{n+1}{n+m+1}$$

then the quadratique form $\mathcal{Q}(S_0, S_n)$ is positive.

Proof. B is positive, and because

$$\frac{2}{n+m+1} < \alpha \leq \frac{n+1}{n+m+1}$$

A is positive according to lemma 3.4. Recall the condition for C to be positive given by lemma 3.3 :

$$\frac{2}{m+n+1} < \alpha \leq \frac{2(m^2-2)}{(n+m+1)(2m-3)}$$

Let's prove that :

$$\frac{n+1}{m+n+1} \leq \frac{2(m^2-2)}{(n+m+1)(2m-3)}$$

if $1 \leq m$.

$n+m+1$ is positive, this implies to prove :

$$n+1 \leq \frac{2(m^2-2)}{2m-3}$$

Because $n \leq m$ we have $n+1 \leq m+1$ Let's prove that

$$n+1 \leq m+1 \leq \frac{2(m^2-2)}{2m-3}$$

We suppose that $1 < m$ then $2m-3$ is positive and the condition is equivalent to :

$$(m+1)(2m-3) \leq 2(m^2-2)$$

This is equivalent to

$$0 \leq 2(m^2-2) - (m+1)(2m-3) = m-1$$

. This is true because $1 < m$. □

The theorem 3.1 and lemma 3.2 implie the following corollary :

Corollary 3.1. *If $1 < n \leq m$, then in a double star network it is possible to get a fairness index α such that*

$$\alpha \leq \frac{n+1}{n+m+1}$$

If $n > m > 1$ then we set $Y_0 = X_n, Y_1 = X_{n+1}, \ldots, Y_{p-1} = X_{n+m}$, $Y_p = X_0, Y_{p+1} = X_1, \ldots Y_{p+q} = X_{n-1}$ this gives a $SN_{p,q}$ double star network with $p = m+1$ and $q = n-1$. Notice that $p < q$ and we prove the following corollary :

Corollary 3.2. *If $1 < m < n$ then in a double star network it is possible to get a fairness index α such that*

$$\alpha \leq \frac{m+2}{n+m+1}$$

3.2.3 Maximal fairness index for double star network.

The corollaries 3.1 and 3.2 give a upper bound for the fairness index of a double star network. This is shown in the next lemma :

Lemma 3.5. *Let α be a fairness index of a double star network $SN_{n,m}$ which has no zero data rate reception.*

- *If $1 < n \leq m$ then $\alpha \leq \frac{n+1}{n+m+1}$ exists*
- *If $1 < m < n$ then $\alpha \leq \frac{m+2}{n+m+1}$ exists*

To validate the theoretical analysis, the next section will present some simulations.

3.3 Double star network simulations

NS2-2.33 is used for the next simulations. CBR over UDP traffic is used. The nodes $X_i, 1 \leq i \leq n-1$ and $X_j, n+1 \leq j \leq n+m$ have a CBR rate of 0.5Mb. The nodes X_0 and X_n have a CBR rate going for 0.1Mbps to 1.0Mbps with a step of 0.5Mbps. The Ad-Hoc routing protocol DSDV is active. The CBR traffic goes from X_0 to $X_i, 1 \leq i \leq n$, for X_n to $X_j, n+1 \leq l \leq n+m$ and backwards.

3.3.1 The $SN_{3,8}$ double star network.

In this case, $n = 3$, $m = 8$ and $n < m$. The lemma 3.5 shows that there can exist fairness index α such that

$$\alpha \leq \frac{1}{3}$$

We do simulation for 1000 time slots, and we get the following results show in this figure 3.3.1 :

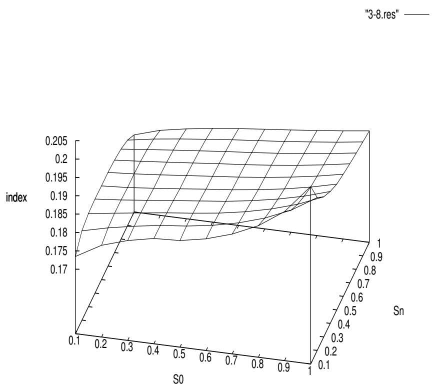

Figure 2. $SN_{3,8}$ fairness index

We notice that the maximal fairness index is given for $S_0 = 1.0Mbps$ and $S_n = 0.1Mbs$. Its value is 0.2008 witch is in the range given by lemma 3.5.

3.3.2 The $SN_{8,3}$ double star network.

In this case, $n = 8$, $m = 3$ and $n > m$. The lemma 3.5 shows that there can exist fairness index α such that

$$\alpha \leq \frac{5}{12}$$

We do simulation for 1000 time slots, and we get the following results show in this figure 3.3.2 :

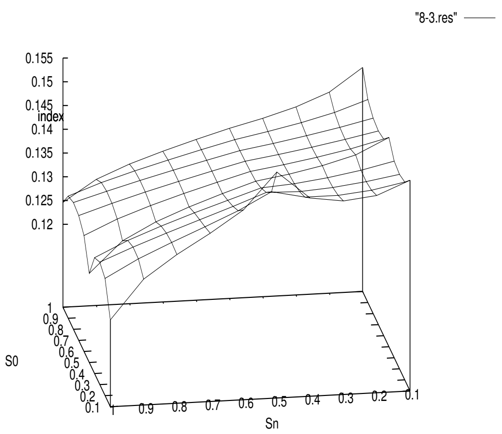

Figure 3. $SN_{8,3}$ **fairness index**

We notice that the maximal fairness index is given for $S_0 = 0.5Mbps$ and $S_n = 0.5Mbs$. Its value is 0.1501 witch is lower than $\frac{1}{6} = \frac{2}{n+m+1}$.

If α will be greater than $\frac{1}{6}$, then the equation 3 will be the equation of an ellipse with variable X and Y. It will be a very lucky to have X, Y on this ellipse. If the fairness index is lower than $\frac{1}{6}$ then equation 3 is easier to solve.

We have given results of existence of fairness index if $\alpha \leq \frac{n+1}{n+m+1}$ or if $\alpha \leq \frac{m+2}{n+m+1}$, but what happens if the fairness index is greater as this values, say $\alpha = 1$. The next section will give an answer in this case.

3.4 Fairness for double star network.

A network is fair if the fairness index is 1. This was studied in the conference [15], and we recall the main results in this section.

To be fair in a double star network, the fairness index of the network must be 1. This implies the following lemma :

Lemma 3.6. *A double star network $SN_{n,m}$ is fair if and only if :*

$$n\,(a - X - Y)^2 + m\,(a + Y)^2 + m(n-1)a^2 = 0 \quad (5)$$

where

- $a = S_0 - S_n$
- $X = \sum_{i=1}^{n-1} S_i$
- $Y = \sum_{i=n+1}^{n+m} S_i$

Proof. If the network is fair, the fairness index is

$$f(x) = 1$$

this gives :

$$\left(\sum_{i=1}^{n} S_i + nS_0 + \sum_{i=n+1}^{n+m} S_i + S_0 + mS_n\right)^2 = (n+m+1)\times$$
$$\left(\left(\sum_{i=1}^{n} S_i\right)^2 + (n+1)S_0^2 + \left(\sum_{i=n+1}^{n+m} S_i\right)^2 + mS_n^2\right)$$

By direct computation, we get the condition (5). □

Remark 2. In equation (5), we can observe that all terms are positive or null. This implies that each term has to be zero for the relation to be validate. We have to discuss about the value of n and m.

3.4.1 Case $n \neq 0, m = 0$

If $m = 0$ and $n \neq 0$, the relation (5) becomes :

$$n(a - X)^2 = 0 \quad (6)$$

This implies that $a = X$. This is the result given for a star network in the paper [14].

3.4.2 Case $n = 0, m \neq 0$

If $n = 0$ and $m \neq 0$ the relation (5) becomes :

$$(a + Y)^2 - a^2 = 0 \quad (7)$$

This implies that $Y = 0$ or $Y = -2 * a$. Because $n = 0$ we have $a = 0$ and then $Y = 0$. In this configuration, no node is transmitting.

3.4.3 Case $n = 1, m \neq 0$

If $n = 1$, then $X = 0$, and the relation (5) becomes:

$$(a - Y)^2 + m\,(a + Y)^2 = 0 \tag{8}$$

Because $m \neq 0$, this implies that :

$$\begin{cases} a - Y = 0 \\ a + Y = 0 \end{cases}$$

In this case, we get $Y = 0$ and $a = 0$. Then only X_0 and X_1 are transmitting with the same rate S_0.

3.4.4 Case $n = 1, m = 0$

If $n = 1$, then $X = 0$, and the relation (5) becomes:

$$(a - Y)^2 = 0 \tag{9}$$

This implies that $Y = a$, and then we get :

$$S_0 = \sum_{i=1}^{m+1} S_i$$

This is the result for the star network SN_{m+1}.

3.4.5 Case $n \neq 0, n \neq 1, m \neq 0$

In this case, the equation (5) has no coefficient equal to zero. Then every term should be equal to zero :

$$\begin{cases} a - X - Y = 0 \\ a + Y = 0 \\ a = 0 \end{cases}$$

The only solution is $a = 0, X = 0, Y = 0$ there is no packet transmitted.

The next theorem was proved :

theorem 3.2. *The fairness of packet transmitted in a $SN_{n,m}$ double star network is given by:*

- $S_0 = \sum_{i=1}^{n} S_i$, *if $< m = 0$ and $n > 0$ this is a SN_n star network,*
- $S_1 = \sum_{i=2}^{m+1} S_i$ *if $n = 1$ and $m > 0$, this is a SN_m star network,*

If $m > 0$ and $n > 1$, then transmitting a packet broke the fairness condition.

This theorem shows that exact fairness can't exist in a non degenerate double-star network. In the next section, we recall the results for [14] whitch gives the results in the degenerate case called star network.

4 Star Network

Definition 4.1 (Star network)**.** The star network SN_n is composed of $n + 1$ nodes $\{X_0, X_1, \ldots, X_n\}$ where $\{X_1, \ldots, X_n\}$ are neighbors of the node X_0, and there is no connexion between X_i, X_j for $i, j > 0$.

The next graph shows a star network :

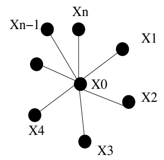

Figure 4. A star network

In this network, the central node X_0 has a degree of n, and each of its neighbors has a degree of 1. This is a very unfair communication channel repartition. When the node X_0 sends a packet, it will be received by the n neighbors. This packet will be seen n times in the network. On the other hand, each packet send by $X_i, i \geq 1$ will be seen only once by X_0. Intuitively, we see that to get fairness, the transmission rate of X_0 must balance the transmission rates of all other nodes. This will be shown in the next section.

4.1 Fairness of the star network

We can compute the fairness conditions for a star network :

Lemma 4.1. *For a star network SN_n, the fairness for the reception rates hold only and only if :*

$$S_0 - \sum_{i=1}^{n} S_i^0 = 0 \tag{10}$$

Proof. Using the expression (2) of x_i, the reception rate of node X_i, we have :

$$f(x) = \frac{\left(\sum_{i=0}^{n} \sum_{j=1}^{D_i} S_j^i\right)^2}{(n+1)\sum_{i=0}^{n} \left(\sum_{j=1}^{D_i} S_j^i\right)^2}$$

If the network is fair, we must have $f(x) = 1$. By a direct computation, we get :

$$S_0 - \sum_{i=1}^{n} S_i^0 = 0$$

□

The relation gives a direct condition on transmission rates to achieve fairness. It is much simpler to compare transmission rates than to evaluate the fairness index. The fairness index is based on a continuous function for transmission rates. This implies that if the difference (10) is close to zero, then the fairness index is close to 1. To get a fair star network, the condition (10) must to move closer. This remark lets us introduce a fairness algorithm which will try to minimize the difference (10) to achieve fairness.

Remark 3. The star network is fair only and only if the transmission rate S_0 is the sum of the transmission rates of all the neighbors of X_0.

4.2 Fairness algorithm

An Ad-Hoc network is seen from a node Y_i as a star network SN_d where d is the degree of the node Y_i. Following (3), we can imagine that the node Y_i adjusts its transmission rate such that it corresponds to the sum of the reception rates. It gets from its neighbors. This gives us the following algorithm running on each node Y_i and using a parameter s given by the administrator of the network :

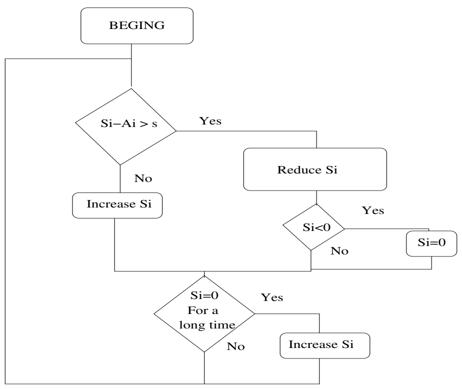

Figure 5. Algorithm

1. Computes the sum of the neighbors transmission rates A_i
2. Compares the sum A_i to the node Y_i transmission rate S_i.
3. If $S_i - A_i > s$ then reduce S_i, if S_j becomes negative, then set it to 0.
4. If $S_i - A_i < s$ then increase S_i if it is possible.
5. If $S_j = 0$ for a long time, then increase it.
6. Go to step 1

This algorithm acts only on the transmission rates. It tries to adjust the difference $S_i - A_i$ to be close to zero. To do this, it needs to have some control over S_i. For a star network, the theoretical approach shows that minimizing the difference $S_i - A_i$ will increase fairness. But it can also be used on any Ad-Hoc networks.

The parameter s controls the sensibility of the algorithm to the standard access algorithm. If s is null, the algorithm will try to always get an exact fairness. This is not realistic, and this can decrease the performance of the network. If s is too high, then the algorithm will have no influence on fairness.

4.3 Simulations

We use the network simulator ns2 to do the simulations. The DSDV routing protocol is used. First, we will do the simulation with the original ns2. After that, we modify ns2 to simulate our algorithm. The transmission rate will be computed on TCP packets sent by each nodes. We use FTP agent to simulate traffic.

4.3.1 The star network SN_6

We will now use SN_6 star network for simulation where two FTP connections (up and down) are established between X_0 and $X_i, i > 0$. The following graph shows the fairness index in a function of time :

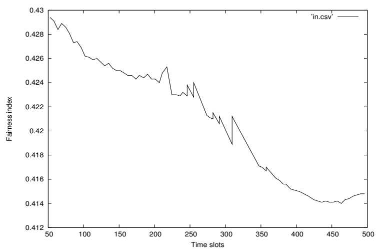

Figure 6. Fairness of a SN_6 star network

We can also compute the difference (10) in function of time slots :

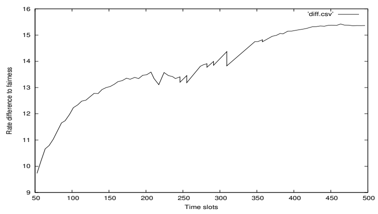

Figure 7. Rate difference to fairness

We notice that the difference (10) increases in time, which is coherent with the fact that the fairness index is decreasing. The aim of our algorithm is to keep the difference (10) close to zero.

We apply our algorithm to this network with $s = 500$. To reduce S_j the algorithm changes the rate from 1Mb to 0.5Mb. When the delay becomes too long, the algorithm reset the node in the standard rate.

This gives us the following graphs :

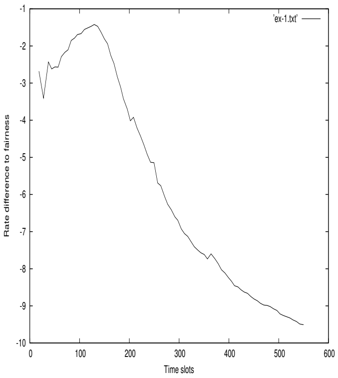

Figure 8. Fairness of a SN_6 star network with modified access algorithm

Remark that the fairness index goes to 0.43 which is better than the simulation done by the default algorithm of ns2.

We can also compute the difference (10) in function of time slots. This is shown in the next graph, see how the algorithm acts on to minimize the difference.

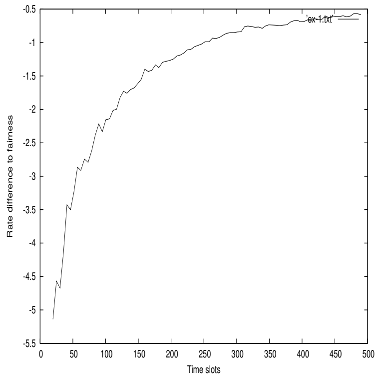

Figure 9. Rate difference to fairness

We notice that the difference rate (10) goes from 15 to -10 which is closer to zero. This improves the fairness index from 0.41 to 0.43. The fairness index is still far from 1, but we can expect that it will be better if the simulation time goes to infinity as described in the following figure.

4.3.2 The star network SN_8

We will now use the SN_8 star network for simulation where two FTP connections (up and down) are established between X_0 and $X_i, i > 0$. The following graph shows the fairness index in a function of time :

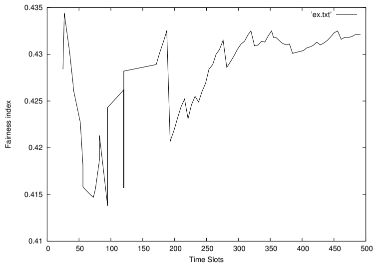

Figure 10. Fairness of a SN_8 star network

We can also compute the difference (10) in function of time slots :

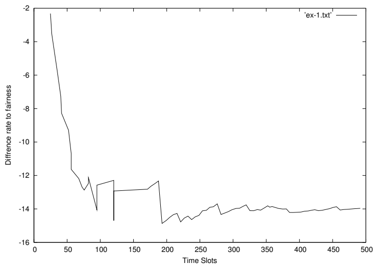

Figure 11. Rate difference to fairness

We can notice that the fairness index is around 0.432 at time slot 500 and the difference (10) is around -14. This confirms that for the star network SN_8, the behavior is not fair. Now we will try to see what is happening with our algorithm. The fairness index is shown in the following graph :

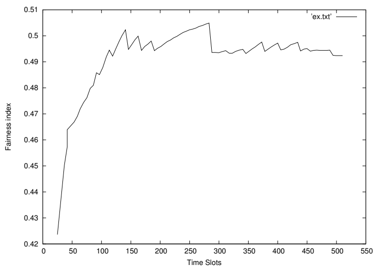

Figure 12. Fairness of a SN_8 star network

This shows that the fairness index is better than the ns2 standard case. At 500, the fairness index is higher than 0.49

and is still increasing. Therefore the algorithm gives better results. Let's take a look at the difference of rate to fairness given by (10) :

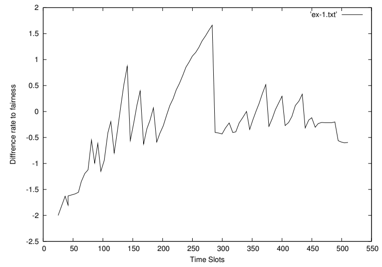

Figure 13. Difference of rate to fairness of a SN_8 star network

This graph shows that the algorithm is working well. The difference is less that -0.6 at time slot 500. The algorithm seems to be efficient.

4.4 A no star network

In this example, the algorithm is applied to no star network. The topology of the network is given by the graph :

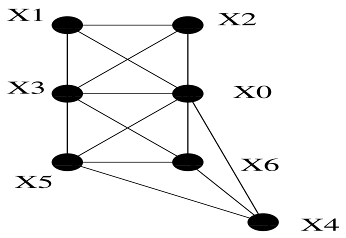

Figure 14. A no star network

There is an FTP traffic simulated for each node to its neighbors. When we applied the standard ns2 simulator, this gave for the fairness index the following result:

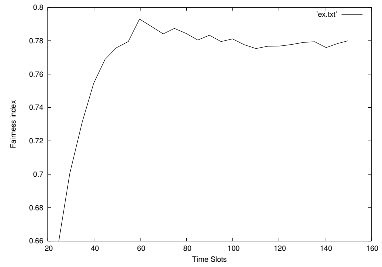

Figure 15. Fairness of the no star network

We can see that the node X_0 is connected to every other node in the network. This node can play the same role as the central node for a star network. The rate difference (10) can be evaluated for this node. This gives the next graph :

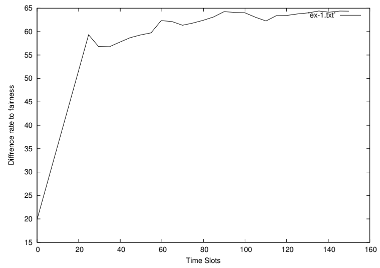

Figure 16. Difference rate of the no star network

The difference is increasing according to the fairness index of the network. This let us supposes that the fairness index is related to the rate difference of X_0 to its neighbors. The algorithm for star networks can be apply to reduce the rate difference (10). When the algorithm is used, the rate difference (10) reacts as following graph :

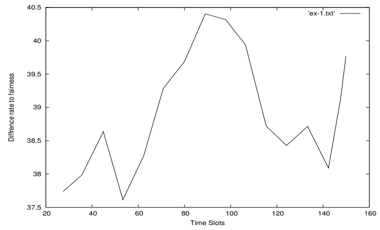

Figure 17. Difference rate of the no star network with our algorithm

The rate difference goes down from 65 to less than 40. The fairness index is shown in the next graph :

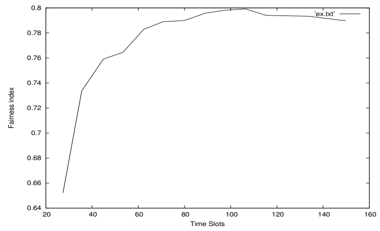

Figure 18. Fairness index of the no star network with algorithm running

Thus the fairness index tends to be closer to 0.8.

In this example, we applied our algorithm to a no star network. The algorithm improves the fairness index.

5 Conclusion

In this work, the study is focused on the fairness of the reception rate. After some generalities, a double star network and a single star network are introduced. This network enables us to compute the fairness index. For double star network, we give some upper bound on fairness index to guaranty their existence. We prove that fairness can only exist if double star network degenerates in star network. Fairness is studied in star networks. Then we elaborate an algorithm to get fairness. The algorithm needs only to know the reception rate and the transmission rate of the node and it can be used on every Ad-Hoc network. But in this case the influence on the fairness index is not developed. Never less and example are shown where the algorithm improves fairness.

The simulation shows that the fairness index is improved for a star network if we apply our algorithm. But the fairness index doesn't seem to react very efficiently. We can expect better results if the simulation time goes to infinity.

In some further work, we propose to apply our algorithm to more complex networks to approach a general Ad-Hoc network. A first step is to compute the maximal fairness index for some topology and then we expect to modify our algorithm to reach this maximal fairness index for a more general Ad-Hoc network.

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